USE OF COMPLEX FUZZY MATRICES IN THE STUDY OF SMOKING PROBLEM

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Abstract. Smoking is one of the dangerous issues in our society. Smoking often enslaves our youth. It then becomes a habit that is easy to start but hard to break. We know that there are various reasons for the cigarette smoking habits of different age groups. Therefore, through this paper we present a new fuzzy mathematical method to identify the main reason for the starting of cigarette smoking in each age, using the concept of complex fuzzy matrices and data. This method will help the country to identify the major cause factor of smoking habit of various age groups and to adopt suitable ways to solve the issue. We apply the concept of relation matrices and modify the algorithm to increase accuracy in the analysis process. An example is also illustrated to verify the developed procedure.

Keywords: - Complex fuzzy sets; Complex fuzzy matrices ; Relation matrix; Comparison matrix; Smoking habit.

2010 Mathematics Subject Classification: - 15B15; 03E72

Introduction

Zadeh [1] was the founder of the concept of fuzzy set theory about 1965. The most important and interesting areas of applications of the theory of fuzzy set are the field of medicine and treatment. Fuzzy matrices are now very rich topic in modeling situations including uncertainty occurred in science, treatments, medical diagnosis, automata etc. Fuzzy matrices were introduced by Thomson [2] in 1977 and these concepts was developed by Kim and Roush. K. T Atanassov was the founder of the notion intuitionistic fuzzy sets, which providing a flexible model to elaborate uncertainty and vagueness in decision making problems. The concept of fuzzy matrices is used in almost all the branches of science. Fuzzy matrices are a better implement for modelling different problems occurring in uncertain situations in different fields of science like computer science, robotics, medical science, artificial intelligence and may others. In 2002, Ramot et. al [3] defined the Complex Fuzzy sets as a generalization of fuzzy sets whose co domain is not restricted to [0, 1] but it is expanded on the unit disc in the complex plane (the set of all complex numbers with modulus less than or equal to 1). Ramot [3] used the idea of complex degree of membership in polar coordinating, where the amplitude is the degree of an object of the Complex Fuzzy Set and the role of phase is to add information which is related to spatial or temporal periodicity of the specific fuzzy set. In 2015, Zhi- Quig Zaho and Shong-Quan Ma, were introduced the concept of complex fuzzy matrices. Fuzzy mathematics is a powerful tool for modelling medical diagnosis process and analysis of various decision-making problems. By the use of complex fuzzy matrices, we presented a new algorithm which will make the analysis more precise.

Preliminaries

Fuzzy Sets

A fuzzy set is a pair (U, μ) where U is a non-empty set and $\mu: U \to [0,1]$ a membership function. The set U is called the universe of discourse and for each $x \in U$, the value $\mu(x)$ is called the degree of membership of x in (U, μ). Then the function μ is called the membership function of the fuzzy set (U, μ).

For a finite set, $U = \{x_1, x_2, x_3, \dots, x_n\}$, the fuzzy set (U, μ) is often denoted by, $\{\mu(x_1)/x_1, \mu(x_2)/x_2, \mu(x_3)/x_3, .., \mu(x_n)/x_n\}$

Let $x \in U$, then x is called;

- Not included in the fuzzy set (U, μ), if $\mu(x) = 0$.
- Fully included in the fuzzy set (U, μ), if $\mu(x) = 1$.
- Partially included in the fuzzy set (U, μ), if $0 < \mu(x) < 1$.

Fuzzy Matrices

A fuzzy matrix is a matrix which has its elements from the unit interval [0,1], called fuzzy unit interval.

A fuzzy matrix A of order $m \times n$ is defined as $A = \left[\langle a_{ij}, a_{ij\mu}\rangle\right]_{m\times n}$, where $a_{ij\mu}$ is the membership value of a_{ij} in A.

For simplicity, we write A as, $A = \left[a_{ij\mu}\right]_{m\times n}$

Example:

$$A = \begin{bmatrix} 0.5 & 0.1 & 0.7 & 0.5 \\ 0.3 & 0.8 & 0.1 & 0.6 \\ 0.6 & 0.4 & 0.9 & 0.8 \\ 0.2 & 0.7 & 0.3 & 0.4 \end{bmatrix}_{4\times 4} \qquad (1)$$

Remark

When we consider [2] the average sunspot number since 1800. During the minimum activity, there are few sunspots, whereas during the solar maximum activity there are many sunspots. According to Ramot, a complex fuzzy set is used to convey information related to the monthly solar activity as well as its position in the unit circle. Under this formalism of complex fuzzy sets, the position in a cycle is represented by the phase variable, which is a real function, and not a degree of membership and the solar activity for a specific month is represented by a degree of membership in a fuzzy set.

For example, the fuzzy set induced by the assertion, "high solar activity". As noted, the cycle of length is 11 years; it starts from a solar minimum goes through a maximum and end at the next solar minimum. Let 0 be the starting point, let π be the solar maximum and 2π denote the next solar minimum. Suppose the traditional grade of membership of a month M_i in the set "High solar activity" is 0.4, and assume that this month is at the peak of solar activity for the cycle then the grade of membership is denoted by $0.4e^{i\pi}$.

On the other hand, Mj is characterized by the complex grade of membership $0.60.6e^{i\frac{\pi}{2}}$, then it means that M_j is in the increasing process of solar activity and it is medium active.

Complex Fuzzy Matrices

A complex fuzzy matrix is a matrix which has its elements from the unit ball in the complex plane.

A complex fuzzy matrix A of order $m \times n$ is defined as

$$A = \left[\langle a_{ij}, a_{ij\mu}\rangle\right]_{m\times n};\ a_{ij\mu} = r_{ij}e^{i\omega_{ij}}; r_{ij} \in [0,1];\ \omega_{ij} \in [0,2\pi) \quad (2)$$

Example: $A = \begin{bmatrix} 0.7e^{i\pi} & 0.5e^{i\frac{\pi}{2}} & 0e^{i0} \\ 1e^{i0} & 0.2e^{i\pi} & 0.1e^{i\frac{\pi}{4}} \\ 0e^{i0} & 0.3e^{i\frac{\pi}{3}} & 1e^{i0} \end{bmatrix}_{3\times 3}$ (3)

Relative Values

We can say that, the relative values depend on other values. Here we use the following equation to find the relative values from the known values.

$r_{ij} = f\left(\frac{P_i}{d_j}\right); i = 1, 2, 3, \ldots; j = 1, 2, 3, \ldots$ (4)

Comparison Matrix

A comparison matrix helps to compare attributes and characteristics of items and helps us to conclude the comparative and relative study. Here we use the following method to create the comparison matrix using the relative values.

Comparison matrix, $R = [r_{ij}]$; $r_{ij} = f\left(\frac{P_i}{d_j}\right); i = 1, 2, 3, \ldots; j = 1, 2, 3, \ldots$ (5)

STUDY OF SMOKING PROBLEM USING COMPLEX FUZZY MATRICES

Cigarette smoking is one of the dangerous problems with our society. Most people start smoking in their teens. The tobacco industry's advertising, cheapness, and promotions for its products, the influence of friends and parents who use tobacco, and life problems have a major impact on tobacco use in our society. The industry spends billions of dollars each year to create and market advertisements that make smoking to look exciting and attractive. Studies show that anyone that starts using tobacco becomes addicted to nicotine. It causes to life-altering life-threatening diseases in those who use tobacco and those who live with them.

The study has a lot of confounding alternatives. Therefore, the planning of analysis procedure is based on the study of combination of variables with complex outcomes.

At, first, we consider the input variables as the main factors that cause cigarette smoking with their membership functions. In second step, we use the concept of complex fuzzy sets and complex fuzzy matrices to identify the major cause of smoking in the corresponding age group.

The input variables are: -

1. Parental influences -d_1
2. Social rewards-d_2
3. Risk taking behavior-d_3
4. Self-medication-d_4
5. Advertising/ Media influence-d_5

On the basis of above-mentioned five input variables and by the use of survey result of 150 peoples, 30 from each age groups, we have obtained the following reports.

The division of age groups as follows;

A_1: 14-17

A_2: 18-22

A_3: 23-27

A_4: 28-32

A_5: 33-36

.	d_1	d_2	d_3	d_4	d_5
A_1	28	13	25	10	26
A_2	25	22	29	23	29
A_3	20	25	26	25	25
A_4	16	28	21	29	20
A_5	10	30	16	28	18

Table 1

Parental Influence- d_1

There is a close connection with the smoking habit of parents and their children smoking. Children of active smokers are more interested to start smoking than the children of non- smokers.

From the survey result,

When d_1 range is $0 \leq d_1 \leq 10$, means that the parents have lower influence in the smoking habit of the corresponding age group. Then, we use the phase value 0 to represent the range in the complex fuzzy set.

When d_1 range is $10 < d_1 < 20$, means that the parents have medium influence in the smoking habit of the corresponding age group, we use the phase value $\frac{2\pi}{3}$to represent the range in the complex fuzzy set.

When d_1 range is $20 \leq d_1 \leq 30$, means that the parents have higher influence in the smoking habit of the corresponding age group, we use the phase value $\frac{4\pi}{3}$ to represent the range in the complex fuzzy set.

The defined membership functions are,

$$\mu_{d_1}(x) = \begin{cases} 0 & ; 0 \leq x \leq 10 \\ \frac{20-x}{10} & ; 10 < x < 20 \\ 0 & ; 20 \leq x \leq 30 \end{cases} \quad (6)$$

$$= \begin{cases} 0e^{i0} \\ \frac{20-x}{10} e^{i\frac{2\pi}{3}} \\ 1e^{i\frac{4\pi}{3}} \end{cases} \quad (7)$$

Social Rewards- d_2.

Social rewards are rewarding that people feel they receive when they participate in a group activity

From the survey result,

When d_2 range is $0 \leq d_2 \leq 10$, means that the social rewards have lower influence in the smoking habit of the corresponding age group. Then, we use the phase value 0 to represent the range in the complex fuzzy set.

When d_2 range is $10 < d_2 < 20$, means that the social rewards have medium influence in the smoking habit of the corresponding age group, we use the phase value $\frac{2\pi}{3}$to represent the range in the complex fuzzy set.

When d_2 range is $20 \leq d_2 \leq 30$, means that the social rewards have higher influence in the smoking habit of the corresponding age group, we use the phase value $\frac{4\pi}{3}$ to represent the range in the complex fuzzy set.

The defined membership functions are,

$$\mu_{d_2}(x) = \begin{cases} 0 & ; 0 \le x \le 10 \\ \frac{20-x}{10} & ; 10 < x < 20 \\ 0 & ; 20 \le x \le 30 \end{cases} \quad (8)$$

$$= \begin{cases} 0e^{i0} \\ \frac{20-x}{10} e^{i\frac{2\pi}{3}} \\ 1e^{i\frac{4\pi}{3}} \end{cases} \quad (9)$$

Risk Taking behavior- d_3

There are many general restrictions on their ability to smoke when they want, such as rules and designated smoking areas. But almost all young people tend towards the risk-taking behavior of smoking.

From the survey result,

When d_3 range is $0 \le d_3 \le 10$, means that the risk-taking behavior have lower influence in the smoking habit of the corresponding age group. Then, we use the phase value 0 to represent the range in the complex fuzzy set.

When d_3 range is $10 < d_3 < 20$, means that the risk-taking behavior have medium influence in the smoking habit of the corresponding age group, we use the phase value $\frac{2\pi}{3}$to represent the range in the complex fuzzy set.

When d_3 range is $20 \le d_3 \le 30$, means that the risk-taking behavior have higher influence in the smoking habit of the corresponding age group, we use the phase value $\frac{4\pi}{3}$ to represent the range in the complex fuzzy set.

The defined membership functions are,

$$\mu_{d_3}(x) = \begin{cases} 0 & ; 0 \le x \le 10 \\ \frac{20-x}{10} & ; 10 < x < 20 \\ 0 & ; 20 \le x \le 30 \end{cases} \quad (10)$$

$$= \begin{cases} 0e^{i0} \\ \frac{20-x}{10} e^{i\frac{2\pi}{3}} \\ 1e^{i\frac{4\pi}{3}} \end{cases} \quad (11)$$

Self-medication-d_4

Some people find cigarette smoking is the best self-medication for relief from some mental illness and some mental stress.

From the survey result,

When d_4 range is $0 \le d_4 \le 10$, means that the self-medication plays lower influence in the smoking habit of the corresponding age group. Then, we use the phase value 0 to represent the range in the complex fuzzy set.

When d_4 range is $10 < d_4 < 20$, means that the self-medication plays medium influence in the smoking habit of the corresponding age group, we use the phase value $\frac{2\pi}{3}$to represent the range in the complex fuzzy set.

When d_4 range is $20 \le d_4 \le 30$, means that the self-medication plays higher influence in the smoking habit of the corresponding age group, we use the phase value $\frac{4\pi}{3}$ to represent the range in the complex fuzzy set.

The defined membership functions are,

$$\mu_{d_4}(x) = \begin{cases} 0 & ; 0 \leq x \leq 10 \\ \frac{20-x}{10} & ; 10 < x < 20 \\ 0 & ; 20 \leq x \leq 30 \end{cases} \quad (12)$$

$$= \begin{cases} 0e^{i0} \\ \frac{20-x}{10} e^{i\frac{2\pi}{3}} \\ 1e^{i\frac{4\pi}{3}} \end{cases} \quad (13)$$

Advertising/ Media influence- d_5

Media and advertisement highly influence the viewers decision making compacity. Similar way, smoking habit also can be influenced by media and advertisements.

From the survey result,

When d_5 range is $0 \leq d_5 \leq 10$, means that the media and advertisements play lower influence in the smoking habit of the corresponding age group. Then, we use the phase value 0 to represent the range in the complex fuzzy set.

When d_5 range is $10 < d_5 < 20$, means that the media and advertisements play medium influence in the smoking habit of the corresponding age group, we use the phase value $\frac{2\pi}{3}$ to represent the range in the complex fuzzy set.

When d_5 range is $20 \leq d_5 \leq 30$, means that the media and advertisements play higher influence in the smoking habit of the corresponding age group, we use the phase value $\frac{4\pi}{3}$ to represent the range in the complex fuzzy set.

The defined membership functions are,

$$\mu_{d_5}(x) = \begin{cases} 0 & ; 0 \leq x \leq 10 \\ \frac{20-x}{10} & ; 10 < x < 20 \\ 0 & ; 20 \leq x \leq 30 \end{cases} \quad (14)$$

$$= \begin{cases} 0e^{i0} \\ \frac{20-x}{10} e^{i\frac{2\pi}{3}} \\ 1e^{i\frac{4\pi}{3}} \end{cases} \quad (15)$$

Remark

In all the membership functions of each fuzzy sets mentioned above, we introduce the phase value to represent the position of that particular input variable in that fuzzy set along with its degree of membership. So that, all the above defined fuzzy sets become complex fuzzy sets. This aspect helps us to use the concept of complex fuzzy matrices and which is very helpful in the further developments of decision supporting system.

From Table 1, and using the above-mentioned membership function, we get,

The first-row matrix for the age group A_1 is,

$$A_1 \to \begin{bmatrix} 1e^{i\frac{4\pi}{3}} \\ 0.7e^{i\frac{2\pi}{3}} \\ 1e^{i\frac{4\pi}{3}} \\ 0e^{i0} \\ 1e^{i\frac{4\pi}{3}} \end{bmatrix}^T \quad (16)$$

The second-row matrix for the age group A_2 is,

$$A_2 \rightarrow \begin{bmatrix} 1e^{i\frac{4\pi}{3}} \\ 1e^{i\frac{4\pi}{3}} \\ 1e^{i\frac{4\pi}{3}} \\ 1e^{i\frac{4\pi}{3}} \\ 1e^{i\frac{4\pi}{3}} \end{bmatrix}^T \qquad (17)$$

The third-row matrix for the age group A_3 is,

$$A_3 \rightarrow \begin{bmatrix} 1e^{i\frac{4\pi}{3}} \\ 1e^{i\frac{4\pi}{3}} \\ 1e^{i\frac{4\pi}{3}} \\ 1e^{i\frac{4\pi}{3}} \\ 1e^{i\frac{4\pi}{3}} \end{bmatrix}^T \qquad (18)$$

The fourth-row matrix for the age group A_4 is,

$$A_4 \rightarrow \begin{bmatrix} 0.4e^{i\frac{2\pi}{3}} \\ 1e^{i\frac{4\pi}{3}} \\ 1e^{i\frac{4\pi}{3}} \\ 1e^{i\frac{4\pi}{3}} \\ 1e^{i\frac{4\pi}{3}} \end{bmatrix}^T \qquad (19)$$

The fifth-row matrix for the age group A_5 is,

$$A_5 \rightarrow \begin{bmatrix} 0e^{i0} \\ 1e^{i\frac{4\pi}{3}} \\ 0.4e^{i\frac{2\pi}{3}} \\ 1e^{i\frac{4\pi}{3}} \\ 0.2e^{i\frac{2\pi}{3}} \end{bmatrix}^T \qquad (20)$$

So, the Age group- Cause complex fuzzy matrix is,

$$\begin{bmatrix} 1e^{i\frac{4\pi}{3}} & 0.7e^{i\frac{2\pi}{3}} & 1e^{i\frac{4\pi}{3}} & 0e^{i0} & 1e^{i\frac{4\pi}{3}} \\ 1e^{i\frac{4\pi}{3}} & 1e^{i\frac{4\pi}{3}} & 1e^{i\frac{4\pi}{3}} & 1e^{i\frac{4\pi}{3}} & 1e^{i\frac{4\pi}{3}} \\ 1e^{i\frac{4\pi}{3}} & 1e^{i\frac{4\pi}{3}} & 1e^{i\frac{4\pi}{3}} & 1e^{i\frac{4\pi}{3}} & 1e^{i\frac{4\pi}{3}} \\ 0.4e^{i\frac{2\pi}{3}} & 1e^{i\frac{4\pi}{3}} & 1e^{i\frac{4\pi}{3}} & 1e^{i\frac{4\pi}{3}} & 1e^{i\frac{4\pi}{3}} \\ 0e^{i0} & 1e^{i\frac{4\pi}{3}} & 0.4e^{i\frac{2\pi}{3}} & 1e^{i\frac{4\pi}{3}} & 0.2e^{i\frac{2\pi}{3}} \end{bmatrix}_{5\times5} \qquad (21)$$

Calculate the relative values $r_{ij} = f\left(\frac{P_i}{d_j}\right)$ and form the comparison matrix R.

$$\mathrm{R} = [r_{ij}]_{5\times5} = \left[f\left(\frac{P_i}{d_j}\right)\right]_{5\times5}; i,j = 1,2,3,4,5 \qquad (22)$$

$$\mathrm{r}_{11} = f\left(\frac{P_1}{d_1}\right) = \frac{|\mu_{d_1}(P_1)| - |\mu_{P_1}(d_1)|}{max\{|\mu_{d_1}(P_1)|, |\mu_{P_1}(d_1)|\}} = \frac{1-1}{max\{1,1\}} = \frac{0}{1} = 0 \qquad (23)$$

$$\mathrm{r}_{12} = f\left(\frac{P_1}{d_2}\right) = \frac{|\mu_{d_2}(P_1)| - |\mu_{P_1}(d_2)|}{max\{|\mu_{d_2}(P_1)|, |\mu_{P_1}(d_2)|\}} = \frac{0.7-1}{max\{0.7,1\}} = \frac{-0.3}{1} = -0.3 \qquad (24)$$

$$\mathrm{r}_{13} = f\left(\frac{P_1}{d_3}\right) = \frac{|\mu_{d_3}(P_1)| - |\mu_{P_1}(d_3)|}{max\{|\mu_{d_3}(P_1)|, |\mu_{P_1}(d_3)|\}} = 0$$

$$r_{14} = f\left(\frac{P_1}{d_4}\right) = \frac{|\mu_{d_4}(P_1)| - |\mu_{P_1}(d_4)|}{max\{|\mu_{d_4}(P_1)|, |\mu_{P_1}(d_4)|\}} = -1$$

$$r_{15} = f\left(\frac{P_1}{d_5}\right) = \frac{|\mu_{d_5}(P_1)| - |\mu_{P_1}(d_5)|}{max\{|\mu_{d_5}(P_1)|, |\mu_{P_1}(d_{15})|\}} = 1$$

$$r_{21} = f\left(\frac{P_2}{d_1}\right) = \frac{|\mu_{d_1}(P_2)| - |\mu_{P_2}(d_1)|}{max\{|\mu_{d_1}(P_2)|, |\mu_{P_2}(d_1)|\}} = 0.3$$

$$r_{22} = f\left(\frac{P_2}{d_2}\right) = \frac{|\mu_{d_2}(P_2)| - |\mu_{P_2}(d_2)|}{max\{|\mu_{d_2}(P_2)|, |\mu_{P_2}(d_2)|\}} = 0$$

$$r_{23} = f\left(\frac{P_2}{d_3}\right) = \frac{|\mu_{d_3}(P_2)| - |\mu_{P_2}(d_3)|}{max\{|\mu_{d_3}(P_2)|, |\mu_{P_2}(d_3)|\}} = 0$$

$$r_{24} = f\left(\frac{P_2}{d_4}\right) = \frac{|\mu_{d_4}(P_2)| - |\mu_{P_2}(d_4)|}{max\{|\mu_{d_4}(P_2)|, |\mu_{P_2}(d_4)|\}} = 0$$

$$r_{25} = f\left(\frac{P_2}{d_5}\right) = \frac{|\mu_{d_5}(P_2)| - |\mu_{P_2}(d_5)|}{max\{|\mu_{d_5}(P_2)|, |\mu_{P_2}(d_5)|\}} = 0$$

Similarly, we get all the remaining relative values,

$r_{31} = 0;\ r_{32} = 0;\ r_{33} = 0;\ r_{34} = 0;\ r_{35} = 0.6$

$r_{41} = 1;\ r_{42} = 0;\ r_{43} = 0;\ r_{44} = 0;\ r_{45} = 0$

$r_{51} = -1;\ r_{52} = 0;\ r_{53} = -0.6;\ r_{54} = 0;\ r_{55} = 0$

So, the comparison matrix is,

$$\begin{bmatrix} 0.0000 & -0.300 & 0.0000 & -1.000 & 1.0000 \\ 0.3000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.6000 \\ 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ -1.000 & 0.0000 & -0.600 & 0.0000 & 0.0000 \end{bmatrix}_{5\times5} \quad (25)$$

From the ranking of the problem, conclude that the main cause of smoking habit in age group A_1 is d_5. That is, the media and advertisements, for the age group A_2 the content d_1. That is, the parental influence, for the age group A_3 the content d_5. That is, the content media and advertisement, for the age group A_4 the content d_1. That is, the parental influence and finally for the age group A_5 the contents d_2, d_4, d_5. That is, social rewards, self-medication and media/advertisements.

CONCLUSION

In this paper, we use the concept of complex fuzzy matrices. The concept of complex fuzzy sets has undergone an evolutionary process since they first introduced. Medical field is the best field in which the consent of fuzzy sets is applicable. The theory of fuzzy matrices in the field of human disease diagnosis was recognized quite early. The doctor generally gathers knowledge about the patient from the past history and laboratory test results. The knowledge provided by each of these factors carries with it varying degrees of uncertainty. This problem can be overcome using the concept of complex fuzzy matrices.

Hence in this paper, by the use of the newly introduced concept of complex fuzzy metrics, we identify the main cause of smoking habits in different age groups. The method assumes knowledge of risk factors affecting the different ages. This method will help the analyst and the country to identify the main problems faced by different age groups of our teens and youth. That is, if the risk factor of a particular age group is parental influence, this method will assist the country to save smokers of that particular age group by launching some innovative counselling methods and strategies. With such an approach we can make our youth healthier. In the future this method can be applied to other types of modeling problems and decision making in the treatment of various cancers and issues.

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